A low α_s and its consequences for unified model building

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Abstract

We review various ways of obtaining consistency between the idea of supersymmetric grand unification and an apparent low value of $\alpha_s \sim 0.112$ indicated by several low energy experiments. We argue that to reconcile the low value of α_s with the predictions of supersymmetric GUTs, we need to go beyond the standard minimal supersymmetric GUT scenario and invoke new physics either at $10^{11} - 10^{12}$ GeV, or at the GUT scale.

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1 Introduction

Recently there has been a lot of interest in the difference in the measured values of α_s in high and low energy experiments. A very useful summary of the issues have been given in a recent paper by Shifman[1] suggesting that the observed discrepancy between the higher values of ≈ 0.125 for $\alpha_s(M_Z)$ derived from the global fit of LEP/SLC data assuming only the SM particle content and interactions on the one hand and lower values near ≈ 0.11 derived from the low energy data such as deep inelastic electron scattering [2, 3], lattice calculations [4] involving the upsilon and the J/Ψ system etc on the other should be considered to be an indication of the presence of new physics[5]. If this new physics is identified with the supersymmetric version of the standard model at low energy, then one can attempt to do a global fit to all LEP/SLC data including the supersymmetric particles and interactions and see if indeed the high value of $\alpha_s(M_Z)$ indicated there is lowered. Such an analysis has been carried out recently by several groups[6], who have shown that if the stop and the chargino masses are kept below a 100 GeV, then there are new contributions to the $Z \to b\overline{b}$ decay which increase its decay width. In the presence of these contributions, the global fit to LEP/SLC data indeed leads to a value for $\alpha_s(M_Z) \simeq .112$ which is what the low energy data give for this parameter. Of course it should be noted that the values of parameters used in the above discussion may not be obtainable in simple SUSY GUT theories. It should be noted that there are a number of other suggestions that could also lead to a higher value for the $Z \to b\overline{b}$ width. It could very well be that one of these scenarios rather than the SUSY contribution is at the real heart of the problem. But for our discussion of unification, it important that either the experimental value of the $Z \to b\overline{b}$ come down or that the supersymmetric scenario provide an explanation for its enhancement over the standard model value so that the value of $\alpha_s(M_Z)$ gravitates towards its lower value. As we discuss below, this low value will have profound implications for the nature of SUSY GUT theories.

The LEP measurements of the gauge couplings α_1, α_2 and α_s combined with the coupling constant evolution dictated by the minimal supersymmetric standard model, have in the past three years led to speculations that the present data while providing overwhelming support for the standard model may in fact be indicating that the next level of physics consists of a single scale supersymmetric grand unified theory, with new physics beyond super-

symmetry appearing only at the scale of 10^{16} GeV[7]. This has generated a great deal of excitement and activity in the area of supersymmetric grand unified theories(SUSY GUT). Consequently, the renormalization group (RG) analysis have become more refined over the time including the various low and high scale threshold corrections. The effect of low energy threshold corrections has been included in the simple step function approximation as well as by including a detailed mass dependent renormalization scheme [8, 9, 10]. These analysis conclude that the effect of the low energy threshold effects is to increase the predicted value of α_s. The GUT scale threshold corrections to α_s come mainly from two sources in the minimal scheme- the doublet-triplet splitting and the mass splitting among the heavy scalars present at the GUT scale. The threshold effects from the well known doublet-triplet splitting always increases the prediction of α_s. The threshold effects from the splitting among the heavy scalars can be of either sign depending on the mass spectrum of the heavy scalars. In the minimal SU(5) GUT the heavy scalars reside in the 24 dimensional adjoint representation of SU(5) and the spectrum the masses of these heavy scalars are quite constrained. A combined analysis including the heavy and light threshold corrections in the minimal SU(5) GUT predicts a value of α_s around 0.126[8] when the superpartner masses are at the TeV scale. If the s-particle masses are less than 1 TeV the prediction of α_s increases further. For example, for s-particle masses around 500 GeV, the predicted value of α_s is 0.130, even beyond the value measured at LEP. Clearly, this leads to a conflict. On the one hand to reduce the prediction of α_s in a SUSY GUT one needs a high value of the s-particle masses; on the other hand, to fit the LEP/SLC data with a low α_s in a supersymmetric model, one needs the stop and the chargino below 100 GeV.

In this brief review, we will summarize the arguments leading one to conclude that if the value of the QCD fine structure constant $\alpha_s(M_Z)$ at the weak scale turns out to be around 0.112 as is indicated by several low energy experiments then to reconcile this value with the predictions for supersymmetric GUTs one necessarily needs new physics beyond the usual minimal SUSY GUT scenarios, either at $10^{11} - 10^{12}$ GeV or at the GUT scale. Several new physics possibilities have been identified in recent literature such as, (a) introducing non-universality of gaugino masses at the GUT scale; (b) introducing heavy threshold effects coming from the heavy scalar multiplets at the GUT scale, when the scalar sector of the minimal SUSY SU(5) model is

altered to include 50, $\overline{50}$, and 75 representations of SU(5); (c) introducing a superstring-inspired scalar spectrum in a supersymmetric SO(10) GUT and making room to incorporate an intermediate B-L symmetry breaking scale; (d) introducing an extra mini doublet-triplet splitting near the GUT scale. This paper is organized as follows. In section 2, we state the basic notions; in section 3 we outline how the non-universality of gaugino masses can lead to a lowering of the prediction of α_s; in section 4, we describe the heavy threshold corrections in the SU(5) model leading to the change in the prediction of α_s; in section 5, we describe the introduction of the string inspired scalar spectrum in the supersymmetric SO(10) GUT - and the consequent lowering of the prediction of α_s and in section 6, we describe the extra mini doublet triplet splitting in the GUT scale. In section 7, we present our conclusions.

2   Basic notions

To illustrate our basic procedures, let us consider a toy example in which all the superpartners are degenerate at the scale M_Z, excepting the gluinos and the winos, which are somewhat heavier than the scale M_Z. In such a scenario, the three gauge couplings at the scale M_Z can be related to the unification coupling by the relations,

$$
\begin{aligned}
2\pi\alpha_s^{-1}\left(M_Z\right) &= 2\pi\alpha_U^{-1} + b_s^{susy}\ln\frac{M_U}{M_{\tilde{g}}} + [b_s^{susy} - \Delta_{\tilde{g}}]\ln\frac{M_{\tilde{g}}}{M_Z}, \\
2\pi\alpha_2^{-1}\left(M_Z\right) &= 2\pi\alpha_U^{-1} + b_2^{susy}\ln\frac{M_U}{M_{\tilde{w}}} + [b_2^{susy} - \Delta_{\tilde{w}}]\ln\frac{M_{\tilde{w}}}{M_Z}, \\
2\pi\alpha_1^{-1}\left(M_Z\right) &= 2\pi\alpha_U^{-1} + b_1^{susy}\ln\frac{M_U}{M_Z},
\end{aligned}
\tag{1}
$$

where, b_i^{susy} are the usual one-loop supersymmetric beta function coefficients and $\Delta_{\tilde{g}}$ and $\Delta_{\tilde{w}}$ are the contributions from the gluino and the wino loops to b_s^{susy} and b_2^{susy} respectively and M_U is the grand unification scale. We notice that the quadratic Casimirs of the SU(2), U(1) and SU(3) groups have the values 2,0 and 3 respectively. Using a vector orthogonal to (2,0,3) and (1,1,1) we construct the combination,

$$
c = 2\pi[3\alpha_2^{-1}(M_Z) - \alpha_{1Y}^{-1}(M_Z) - 2\alpha_s^{-1}(M_Z)].
\tag{2}
$$

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Combining Eqn.(1) and Eqn.(2) we have,

$$c = [3b_2^{susy} - b_1^{susy} - 2b_s^{susy}] \ln \frac{M_U}{M_Z} - 3\Delta_{M_{\tilde{w}}} \ln \frac{M_{\tilde{w}}}{M_Z} + 2\Delta_{\tilde{g}} \ln \frac{M_{\tilde{g}}}{M_Z}. \qquad (3)$$

In the right hand side of Eqn.(3) the gauge and the fermionic contributions to the beta function coefficients cancel out due to the orthogonality that has been mentioned earlier, where as the scalar contribution does not. At this stage, using $\Delta_{\tilde{w}} = 4/3$ and $\Delta_{\tilde{g}} = 2$, we have the prediction of α_s, as,

$$\alpha_s^{-1}(M_Z) = \frac{1}{2} [3\alpha_2^{-1} - \alpha_1^{-1}] - \frac{3}{5\pi} \ln \frac{M_U}{M_Z} + \frac{1}{\pi} \ln \frac{M_{\tilde{w}}}{M_Z} - \frac{1}{\pi} \ln \frac{M_{\tilde{g}}}{M_Z}, \qquad (4)$$

where, the second term is the threshold correction due to the doublet triplet splitting, in which M_U and M_Z are the masses of the superheahy triplet and the light doublet higgs scalars. It is interesting to note that the 'gluino'-term and the 'wino'-term in the right hand side of Eqn.(4) have a relative sign among them.

3 Non-universality of gaugino masses

In the minimal scenario the spontaneous breaking of supergravity yields a global supersymmetric theory supplemented by a set of soft supersymmetry-breaking parameters. In particular in SUSY GUT theories, gauge invariance implies that at the GUT scale, one must have a universal gaugino mass $(m_{1/2})$. Using the RG evolution, if one runs the gaugino masses from the GUT scale to the lower scales, a relation between the mass of gluionos and the mass of the winos is gotten [11]. If the masses of the s-particles are near the electroweak scale one gets,

$$x \equiv \frac{m_{\tilde{g}}(M_Z)}{m_{\tilde{w}}(M_Z)} = \frac{\alpha_2(M_Z)}{\alpha_s(M_Z)} \sim 3.3. \qquad (5)$$

This relation is of considerable significance regarding the prediction of α_s in a supersymmetric GUT as we will explain now. To predict α_s we use the complete relation,

$$\begin{aligned}
\alpha_s^{-1}(M_Z) &= \frac{1}{2} [3\alpha_2^{-1}(M_Z) - \alpha_{1Y}^{-1}(M_Z)] - \frac{1}{\pi} \ln \frac{M_{\tilde{g}}}{M_Z} \, \theta(M_{\tilde{g}} - M_Z) \\
&+ \frac{1}{\pi} \ln \frac{M_{\tilde{w}}}{M_Z} \, \theta(M_{\tilde{w}} - M_Z) + T_{heavy} + T_{others}.
\end{aligned} \qquad (6)$$

The theta functions in Eqn.(6) have the value 1 whenever the argument is positive and is zero otherwise. T_{heavy} parametrizes the heavy threshold corrections arising from the doublet-triplet splitting as well as from the splitting among the heavy scalars. T_{others} parametrizes the effect of the light degrees of freedom apart from the winos and the gluinos. We notice that, when the mass of the gluino is less than the mass of Z, neither wino nor the gluino contributes to Eqn(6). When the mass of the gluino crosses the M_Z range, the 'gluino'-term, having a negative sign, starts contributing to the α_s^{-1}, and consequently α_s starts to increase. Only when the mass of the wino becomes comparable to the electroweak scale, the 'wino'-term starts contributing, which compensates the increasing effect. In the Figure 1 [9] this phenomenon is depicted [3]. Notice that the mass of the gluino is always more than the mass of the wino due to the constraining relation given in Eqn.(5).

Clearly, to reverse the effect, one needs to relax the mass relation between the gluino and the wino [12]. This approach has been taken by Roszkowski and Shifman [13]. It has been noted by them that out of all soft masses the soft masses of the wino and the gluino have the dominant influence on the prediction of α_s. The reason for this is two fold,
(a) The soft mass scale of the wino influences only the SU(2) beta function coefficient, whereas the soft mass scale of the gluino influences only the SU(3) beta function coeffcient only.
(b) The contribution of the wino and the gluino in the beta function coefficients of SU(2) and SU(3) groups respectively are the largest among all superparticles.

Along with this observations we may also observe that if the constraint in Eqn.(5) is relaxed [the x parameter is varied], one can alter the prediction of α_s considerably. The two loop predictions of α_s by choosing different values of x have been summarized in Figure 2 [13].

One needs the gluino mass in the ball park of 100 GeV to predict $\alpha_s = 0.11$ by the above mechanism. Such a light gluino will have further phenomenological consequences. For example, the gluino correction enhances the hadronic width of Z. However, the $Z \rightarrow b\bar{b}$ width increases too much for $M_{\tilde{g}} \sim 100$ GeV, and one has to descend to unacceptably low sqark masses to reconcile with the experimental $Z \rightarrow b\bar{b}$ width.

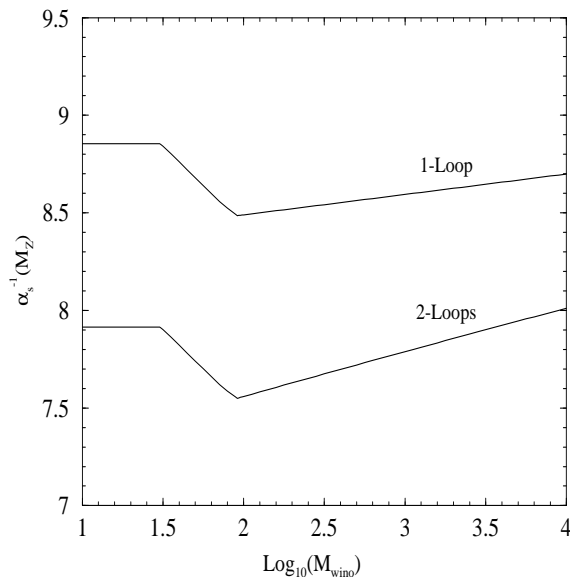[3] There is a 10 % decrease of α_s^{-1} at the 2 loop level.

Figure 1: The one and two loop predictions of $\alpha_s(M_Z)$ in MSSM with universal scalar mass term at the GUT scale.

4 Heavy thresholds and missing doublet SU(5)

In the presence of heavy threshold corrections, the unification scale is no longer well-defined. Let us, therefore, define a scale Λ, which is larger than any GUT scale mass. Above the scale Λ all the couplings remain unified. Again we will consider a toy SU(5) example. Now, as we are interested in the heavy thresholds only, let all the superpartners of the standard model fermions and gauge bosons be degenerate at the scale M_Z. The heavy spectrum of the minimal SU(5) model is simple. The $(3, 2, 5/6) + (\overline{3}, 2, 5/6)$ components of the 24-scalar get absorbed by the heavy gauge bosons with a common mass M_V. The SU(3)-octet and the SU(2)-triplet have a common mass M_Σ whereas the singlet has a mass $0.2M_\Sigma$. We can relate the three gauge couplings at the scale M_Z to the unified coupling at the scale Λ as
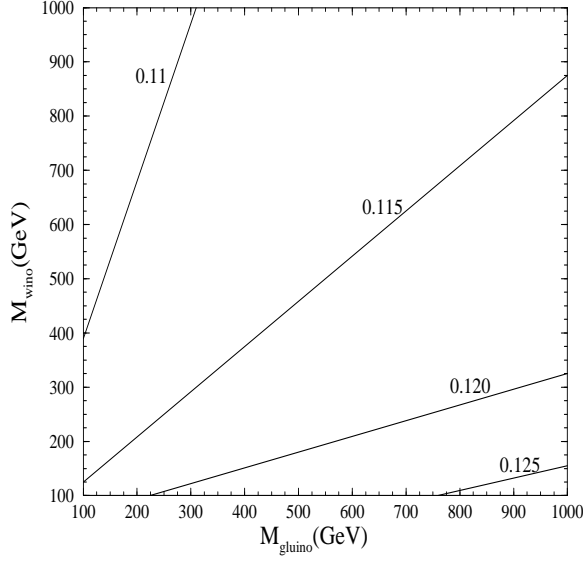
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Figure 2: The contours of constant $\alpha_s(M_Z)$ for various choices of the parameter 'x'.

[30],

$$\alpha_s^{-1}(M_Z) = [\alpha_U^{-1}(\Lambda) - \frac{2}{\pi} \ln \frac{\Lambda}{M_V} + \frac{3}{2\pi} \ln \frac{\Lambda}{M_\Sigma} + \frac{1}{2\pi} \ln \frac{\Lambda}{M_{H_3}}] + \frac{1}{2\pi} b_s^{susy} \ln \frac{\Lambda}{M_Z},$$

$$\alpha_2^{-1}(M_Z) = [\alpha_U^{-1}(\Lambda) - \frac{3}{\pi} \ln \frac{\Lambda}{M_V} + \frac{1}{\pi} \ln \frac{\Lambda}{M_\Sigma}] + \frac{1}{2\pi} b_2^{susy} \ln \frac{\Lambda}{M_Z},$$

$$\alpha_1^{-1}(M_Z) = [\alpha_U^{-1}(\Lambda) - \frac{5}{\pi} \ln \frac{\Lambda}{M_V} + \frac{1}{5\pi} \ln \frac{\Lambda}{M_{H_3}}] + \frac{1}{2\pi} b_1^{susy} \ln \frac{\Lambda}{M_Z}. \qquad (7)$$

Taking the combination c as in the previous section, we find that it is independent of any field that is in the adjoint of SU(5) and we recover the result,

$$\alpha_s^{-1}(M_Z) = \frac{1}{2}[3\alpha_2^{-1} - \alpha_1^{-1}] - \frac{3}{5\pi} \ln \frac{M_{H_3}}{M_Z}. \qquad (8)$$

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Now, it is clear why we interpreted the second term in Eqn. (4) as the effect of doublet triplet splitting.

We notice a special property of the 24-dimensional scalar in relation to the combination c. While the components $(3, 2, 5/6) + (\overline{3}, 2, -5/6)$ are absorbed as the logitudinal components of the heavy SU(5) gauge bosons, the rest are in the adjoint representation of the low energy groups. We have already noticed that fields which are in the adjoint representation of the low energy groups and having a common mass cannot contribute to the combination c. Now, we also note that even the would be goldstone bosons do not contribute to c; and no other component of the adjoint is left; except the singlet which does not contribute to the beta function coefficients. So the splitting in the adjoint 24, does not affect the prediction of α_s at all.

The situation changes if we introduce 75-dimensional scalar instead of the adjoint to break the unification symmetry along with the 50 and $\overline{50}$ dimensional representations needed for the missing doublet mechanism [28]. Twelve components of 75 will be eaten up by the the heavy gauge bosons. Remaining 63 components will be split in mass [29]. Moreover, the color triplets from 50 and $\overline{50}$ Higgs scalars get mixed with the color triplets of 5 and the $\overline{5}$. The rest of the components of 50 and the $\overline{50}$ components are not split among themselves. They acquire a common mass M_{50}. The typical masses of the various GUT scale heavy scalars [29] have been summerized in Table1.

Given the masses and the transformation properties in Table1, it is straightforward to calculate the change in the prediction of $\alpha_s(M_Z)$. Indeed such a calculation have been performed in Ref.[8] in which they have carefully taken into account both the heavy and the light threshold effects. They conclude that a large negative correction to α_s comes in the missing doublet model due to the splitting in the 75 Higgs. The value of $\alpha_s = 0.112$ is achievable (taking into account a lower bound on a combination of M_{D_1} and M_{D_2} from proton decay) with the s-particle masses around the electroweak scale. For further details the reader is referred to their papers.

5   Intermediate scale in supersymmetric SO(10)

There is a general understanding in the literature that the LEP measurements of the gauge couplings at the scale M_Z is a hint to a one step unified

component	mass	comments
$(8,3,0)$	M_Σ	
$(3,1,\pm 5/3)$	$0.8M_\Sigma$	
$(6,2,\pm 5/6)$	$0.4M_\Sigma$	$in\ 75$
$(1,1,0)$	$0.4M_\Sigma$	$of\ SU(5)$
$(8,1,0)$	$0.2M_\Sigma$	
$(3,1,\pm 1/3)$	M_{D_1}	$in\ 5+\bar{5}$
$(3,1,\pm 1/3)$	M_{D_2}	$and\ 50+\overline{50}$
$rest\ of\ 50+\overline{50}$	M_{50}	

Table 1: The representations and masses of the heavy scalars in the missing doublet SU(5) model. Bars have been suppressed in certain representation for the compactness of presentation

theory. There are however several physical arguments based on neutrino physics [15, 16] as well as strong CP problem [14] suggesting that there may be an intermediate scale corresponding to a gauged (local) B-L symmetry breaking somewhere around 10^{11} to 10^{12} GeV. However, such a scale must not affect the gauge unification constraints. To examine this, let us write down the RGE of the three couplings relating them to the gauge coupling at the unification scale M_U to the ones at the weak scale M_Z and introducing a general intermediate scale M_I, as,

$$
\begin{aligned}
2\pi\alpha_1^{-1}(M_Z) &= 2\pi\alpha_U^{-1}(M_U) + b_1\ln\frac{M_I}{M_Z} + b_1'\frac{M_U}{M_I}, \\
2\pi\alpha_2^{-1}(M_Z) &= 2\pi\alpha_U^{-1}(M_U) + b_2\ln\frac{M_I}{M_Z} + b_2'\frac{M_U}{M_I}, \\
2\pi\alpha_s^{-1}(M_Z) &= 2\pi\alpha_U^{-1}(M_U) + b_s\ln\frac{M_I}{M_Z} + b_s'\frac{M_U}{M_I}.
\end{aligned}
\tag{9}
$$

At first, we notice that the one-loop beta function coefficients b_i in MSSM for the groups SU(3), SU(2) and U(1) are -3, 1 and 33/5 respectively. Presently we are interested in the beta function coefficients b_i' governing the slopes in the region between M_I and M_U. So, by taking a vector orthogonal to (-3,1,33/5) and (1,1,1) we construct a second combination c_1 in which b_i's

get eliminated but b'_i s survive, namely,

$$c_1 = 2\pi(7\alpha_s^{-1} - 12\alpha_2^{-1} + 5\alpha_1^{-1}). \tag{10}$$

We combine, Eqn.(9) and Eqn.(10) to get,

$$c_1 = (7b_s - 12b_2 + 5b_1) \ln \frac{M_I}{M_Z} + (7b'_s - 12b'_2 + 5b'_1) \ln \frac{M_U}{M_I}. \tag{11}$$

Due to the orthogonality in the construction of c_1 the coefficient of $\ln(M_I/M_Z)$ vanishes and we are left with,

$$c_1 = (7b'_s - 12b'_2 + 5b'_1) \ln \frac{M_U}{M_I}. \tag{12}$$

Moreover, if we assume that the values of α_1, α_2 and α_s are such that an one-step unification is possible with MSSM and a big desert - we get a condition from Eqn.(11) in the limit $M_U = M_I$, which is,

$$c_1 = 0. \tag{13}$$

Combining, Eqn.(13) and Eqn.(12), we get,

$$(7b'_s - 12b'_2 + 5b'_1) \ln \frac{M_U}{M_I} = 0. \tag{14}$$

We note the two solutions of Eqn.(14).
(a) $M_U = M_I$ which leads us back to the one-step unification.
(b) Due to the presence of new fields above the scale M_I, the beta function coefficients b'_i conspire among themselves to produce[17],

$$7b'_s - 12b'_2 + 5b'_1 = 0. \tag{15}$$

One can restrict the type of Higgs representations above the intermediate scale M_I by requiring that the supersymmetric SO(10) GUT emerges from an underlying superstring theory. If we restrict ourselves to only those Higgs scalars which can arise from simple superstring models with Kac-Moody levels one or two, we can have a restricted number of solutions of Eqn.(15). We will consider an intermediate symmetry group $G_I = U_{B-L} \times SU(2)_L \times SU(2)_R \times SU(3)_C$. The solutions can be characterized by three integers (n_C, n_H, n_X), where n_C

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model	I	II	III	IV	V	VI	VII
(n_C, n_H, n_X)	$(0,1,2)$	$(0,1,3)$	$(0,1,4)$	$(0,2,3)$	$(0,2,4)$	$(0,2,5)$	$(1,2,4)$

Table 2: The various models which satisfy the condition in Eqn. 15 approximately.

refers to the number of (0,1,1,8), n_H means the number of (0,2,2,1) fields and n_X means the number of (1,1,2,1)+(-1,1,2,1) fields under the intermediate symmetry gauge group G_I. The various scenarios[18, 20] are tabulated in Table2.

The predictions of α_s for different models [18, 20] have been plotted in Figure 3. We notice from Figure 3 that the models VI and VII are particularly interesting for the value of α_s in the range of 0.11. A comment is in order. The value of the intermediate scale (M_I) is a new parameter. But we notice that even this new parameter is quite constrained by the the LEP measurements of the couplings at the scale M_Z. Moreover, only three out of the seven intermediate scale models in Table2 will survive if the value of $\alpha_s(M_Z)$ turns to be 0.112.

There is another interesting aspect of such intermediate scale SO(10) GUTs. We see that the requirement of unification forces us to introduce new scalar fields above the scale of $B - L$ symmetry breaking. These new scalars will also affect the evolution of Yukawa couplings above the intermediate symmetry breaking scale. The evolution of various Yukawa couplings have been calculated in Ref [20]. The results are tabulated in Table3 and Table4.

It is widely known that in SUSY GUTs with one step breaking predict a large value of m_b for the major part of the parameter space [21]. The study of b-τ unification including a right handed neutrino has also been performed [22]. However, in this study no new gauge interactions beyond the intermediate scale was considered and due to renormalization effects of the new Yukawa coupling, a 10-15% increase in the mass of the b-quark was obtained. However one sees that the inclusion of the new left-right symmetric gauge and Higgs interactions at M_{B-L} surviving from a string inspired SO(10) GUT, and constrained by Eqn.(15), the running of the b-quark Yukawa coupling is altered and as a result an attractive reconciliation
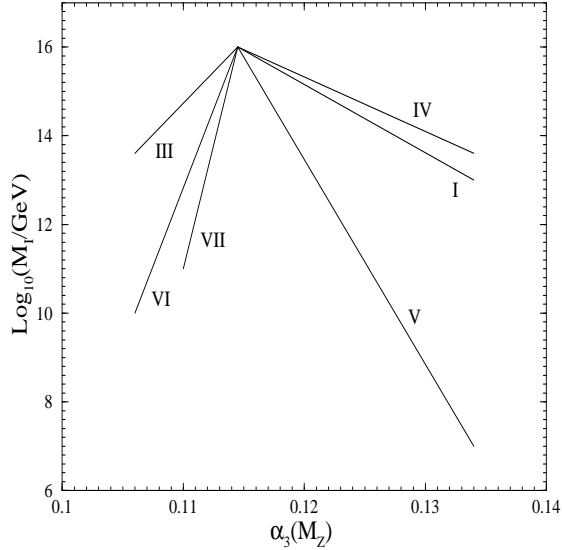
Figure 3: The predictions of α_s are displayed as constant α_s contours for the various models given in Table 2.

with the experimental measurements [4] can be achieved.

6 Mini doublet-triplet splitting

In this section we will explore a possible reverse doublet-triplet splitting [24] which will have an effect opposite to the conventional doublet-triplet splitting on the prediction of α_s. Such a strange reverse doublet-triplet splitting is indeed possible in a SO(10) model when there is a mechanism to strongly suppress the Higgsino mediated proton decay [25] as will be displayed below.

We consider the prediction of α_s including the threshold effects in SUSY SU(5), which is well-studied in the literature [27, 29, 30, 9, 8]. Throughout

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[4] Taking $m_b(m_b)$ in the range 4.1 to 4.5 GeV and $m_\tau(m_\tau)$ to be 1.777 GeV [23], one typically gets $\frac{m_b(M_t)}{m_\tau(M_t)}$ in the range 1.7 to 1.9 [21, 22].

Y_1	Y_2	$h_t(M_t)$	$h_b(M_t)$	$h_\tau(M_t)$	$tan\beta$	$m_b(M_t)$	$m_t(M_t)$	$\frac{m_b(M_t)}{m_\tau(M_t)}$
1	1	1.010	0.96	0.62	60.43	2.77	176.83	1.56
1	10^{-1}	1.060	0.84	0.52	51.26	2.85	184.46	1.60
1	10^{-2}	1.094	0.460	0.270	26.7	3.01	190.34	1.69
1	10^{-3}	1.103	0.160	0.095	9.25	3.06	190.90	1.72
1	10^{-4}	1.104	0.054	0.030	2.80	3.07	181.03	1.73
1	$\frac{1}{2}10^{-4}$	1.104	0.037	0.021	1.85	3.08	169.16	1.73

Table 3: The values of $h_t(M_t)$, $h_b(M_t)$, $h_\tau(M_t)$ and calculated by RGE for $\alpha_s = 0.11$ in model VI. The prediction of the masses m_b and m_t at the scale M_t has been quoted in GeV. M_t is defined as 170 GeV. $tan\beta$ has been calculated assuming $m_\tau(M_Z) = 1.777$ GeV.

this section we will assume that including the threshold corrections, the minimal SUSY SU(5) GUT predicts $\alpha_s = 0.126$ [8]; we will also assume that the mass of the color triplet Higgs scalars in a minimal SU(5) GUT is $10^{16.6}$ GeV [9]. In particular the prediction of α_s in the minimal SUSY SU(5) can be written as,

$$\alpha_s^{-1}(M_Z) = \frac{1}{2}\left[3\alpha_2^{-1}(M_Z) - \alpha_1^{-1}(M_Z)\right] - \frac{3}{5\pi}\ln[\frac{M_3}{M_2}] + T_L, \qquad (16)$$

Where, M_3 and M_2 are the masses of the triplet and the doublet Higgs scalars present in the 5 and $\overline{5}$ representations of SU(5), and T_L parametrizes the contribution from all other light degrees of freedom (excluding the light Higgs doublets) [30], and in a simple step function approximation[5] $T_L = \frac{1}{2\pi}\ln\frac{M_{SUSY}}{M_Z}$. In the minimal model the triplet-mass, which is bounded from below from the non-observation of proton decay, remains at the GUT scale. On the contrary the mass of the doublet is of the order of the electroweak scale. In such a generic situation, that is, whenever $M_3 > M_2$ the doublet-triplet splitting increases the prediction of α_s. However, notice the hypothet-

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[5] M_{SUSY} can be considered in the simplest approach as a common susy breaking scale, or as an effective susy mass parameter [26] resuming the effect of the detailed susy spectrum, and in this sense it can be either more or less than M_Z depending on the super-partner masses.

Y_1	Y_2	$h_t(M_t)$	$h_b(M_t)$	$h_\tau(M_t)$	$tan\beta$	$m_b(M_t)$	$m_t(M_t)$	$\frac{m_b(M_t)}{m_\tau(M_t)}$
1	1	0.99	0.960	0.59	57.81	2.87	173.90	1.61
1	10^{-1}	1.05	0.830	0.50	48.78	2.97	182.65	1.67
1	10^{-2}	1.09	0.460	0.26	25.41	3.17	189.11	1.78
1	10^{-3}	1.10	0.160	0.09	8.81	3.23	189.69	1.82
1	10^{-4}	1.10	0.052	0.023	2.65	3.24	178.80	1.82
1	$\frac{1}{2}10^{-4}$	1.104	0.037	0.020	1.74	3.24	165.70	1.82

Table 4: The values of $h_t(M_t)$, $h_b(M_t)$, $h_\tau(M_t)$ and calculated by RGE for $\alpha_s = 0.11$ in model VII. The prediction of the masses m_b and m_t at the scale M_t has been quoted in GeV. M_t is defined as 170 GeV. $tan\beta$ has been calculated assuming $m_\tau(M_Z) = 1.777$ GeV.

ical possibility, that if the mass of the doublet were more than the mass of the triplet, we would have had a reverse effect on α_s. Keeping this in mind we add one more $5 + \overline{5}$ Higgs scalars with doublet and triplet masses as M_2' and M_3' GeV respectively. In that case the Eqn.(16) gets modified to,

$$\alpha'^{-1}_s(M_Z) = \frac{1}{2}\left[3\alpha_2^{-1}(M_Z) - \alpha_1^{-1}(M_Z)\right] - \frac{3}{5\pi}\ln\left[\frac{M_3 M_3'}{M_2 M_2'}\right] + T_L. \qquad (17)$$

Taking the difference of Eqn.(16) and Eqn.(17) and assuming,

$$M_3 = 10^{16.6} \ ; \ M_2 = 10^2 \ ; \ M_3' = 10^x \ ; \ M_2' = 10^y, \qquad (18)$$

we get,

$$\Delta\alpha_s^{-1} = \alpha'^{-1}_s(M_Z) - \alpha_s^{-1}(M_Z) = \frac{3}{5\pi}(y - x)\ln 10. \qquad (19)$$

It is easy to check from Eqn.(19) that taking $y - x = 2.26$ we can get $\Delta\alpha_s^{-1} = 0.99$ and consequently α_s decreases by 11%, from 0.126 to 0.112. Instead if we add n extra pairs of $5 + \overline{5}$ the required splitting in each SU(5) multiplet is only 2.26/n orders of magnitude.

We can give an example of incorporating this mechanism in a realistic SUSY SO(10) GUT. The main problem to lower the mass of the color triplet Higgs Scalars comes out of the stringent experimental upper bounds imposed on the amplitude of the Higgsino mediated proton decay diagrams.

Babu and Barr [25] have shown that it is possible to suppress the Higgsino mediated proton decay strongly in an SO(10) model by a judicious choice of the fields, couplings and VEVs at the GUT scale. Consider the SO(10) invariant superpotential [25]

$$W = \lambda 10_{1H} 45_H 10_{2H} + \lambda' 10_{2H} 45'_H 10_{3H} + M 10_{3H} 10_{3H} + \sum_{i,j=1}^{3} f_{ij} 16_i 16_j 10_{1H}.$$
(20)

If 45 and 45′ get VEVs in the directions [34, 25]

$$\langle 45 \rangle = \eta \otimes diag(a, a, a, 0, 0) \; ; \; \langle 45' \rangle = \eta \otimes diag(0, 0, 0, b, b) \; ; \; \eta \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$
(21)

the super-heavy mass matrices of the doublets and the triplets are of the form,

$$(\bar{2}_1 \; \bar{2}_2 \; \bar{2}_3) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda'b \\ 0 & -\lambda'b & M \end{pmatrix} \begin{pmatrix} 2_1 \\ 2_2 \\ 2_3 \end{pmatrix} \; \text{and} \; (\bar{3}_1 \; \bar{3}_2 \; \bar{3}_3) \begin{pmatrix} 0 & \lambda a & 0 \\ -\lambda a & 0 & 0 \\ 0 & 0 & M \end{pmatrix} \begin{pmatrix} 3_1 \\ 3_2 \\ 3_3 \end{pmatrix}.$$
(22)

The absence of any direct coupling between $\bar{3}_1$ and 3_1 suppresses the Higgsino mediated proton decay. The absolute values of the masses for the doublets (M_D) and the triplets (M_T) are given by[6],

$$M_D = (0, \frac{M \pm \sqrt{M^2 - \lambda'^2 b^2}}{2}) \; ; \; M_T = (\lambda a, \lambda a, M).$$
(23)

There can be various choices of parameter space leading to the required lowering in the prediction of α_s [see Eqn.(19)]. The simplest choice is, $M^2 = \lambda'^2 b^2$. In this case the masses are,

$$M_D = (0, M/2, M/2) \; \text{and} \; M_T = (\lambda a, \lambda a, M).$$
(24)

We notice that, in this model one pair of doublet-triplet is almost degenerate. Now, using the fact that in minimal SUSY SU(5) the mass of the color triplet is $10^{16.6}$ GeV and using,

$$\lambda a = 10^x \;\; , \;\; M = 10^y,$$
(25)

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[6] When supersymmetry is broken the masses receive correction of the order of $m_{3/2}$.

we get,

$$\Delta\alpha_s^{-1} = \frac{3}{5\pi}[(16.6 - x) + (y - x) - \frac{\ln 4}{\ln 10}] \ln 10. \tag{26}$$

We can achieve the desired suppression of α_s if for example $x = 15.2$ and $y = 16.6$. We expect that such a splitting is not difficult to achieve given the number of parameters in the SO(10) invariant superpotential. The threshold effects due to heavy SO(10) gauge bosons have been discussed in Ref [24].

7   Conclusion

To summarize, we have discussed various ways to reconcile a possible low value of α_s with the idea that all gauge couplings eventually unify . They all indicate new physics beyond the canonical minimal single SUSY GUT scenarios and point towards models with intermediate scales[20, 31] or new Higgs fields at the GUT scale[24] or perhaps even non-GUT type physics[13]. Any realistic model building in the SUSY GUT framework must therefore respect either one of these scenarios. There are of course other suggestions to change the predictions of α_s by including gravitational effects at the GUT scale[32, 33] provided these effects are assumed to occur with an enhanced strength, a possibility which may not be so natural in many theories (although one cannot be completely sure about the strength of the gravitational effects at this stage of our knowledge).

8   Acknowledgements

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